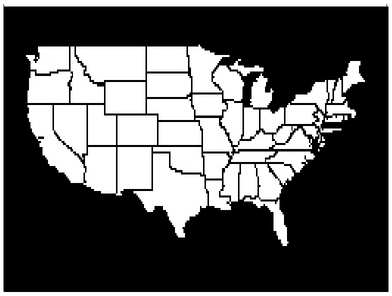

Set Theory
Lesson 5
Set Operations

In order to work with numbers, we use certain operators, such as addition, subtraction, multiplication and division. The same holds true for sets. For sets there are three operators: union, intersection, and complement. With these operators we will develop new sets by forming the union of two or more sets, forming the intersection of two or more sets and taking the complement of a set. This lesson discusses these operators in detail.

5.1 - What is the Union of Two or More Sets?

The union of two or more sets is the collection of all the distinct elements from these sets. This union results in a new set.

The symbol for the union operator is $\cup$.

5.1 - Example 1: If $\mathbf{A}$ = {Mary, Jane, Harry, Bill} and $\mathbf{B}$ = {Jack, Bill, Frankie}, find $\mathbf{A}\cup\mathbf{B}$.

Solution:

$\mathbf{A}\cup\mathbf{B}$ = {Mary, Jane, Harry, Bill, Jack, Frankie}. The union of **A** and **B** is the collection of both sets. Note that the element Bill is in both **A** and **B**. However, for the union to be a set, the element Bill can only appear once.

5.1 - Example 2: If $\mathbf{F} = \{99, -6, 10, 100, 76\}$ and $\mathbf{D} = \{1, 2, 3\}$, find $\mathbf{F}\cup\mathbf{D}$.

Solution:

$\mathbf{F}\cup\mathbf{D} = \{99, -6, 10, 100, 76, 1, 2, 3\}$. Again the union is the collection of the sets **F** and **D**.

5.1 - Example 3: If $\mathbf{F} = \{(a,b), (c,d), (e,f)\}$, $\mathbf{D} = \{(b,a), (e,f)\}$ find $\mathbf{F}\cup\mathbf{D}$.

Solution:

$\mathbf{F}\cup\mathbf{D} = \{(a,b), (c,d), (e,f), (b,a)\}$. Note that (a,b) and (b,a) are distinct elements. Again, the elements of **F** and **D** are collected together to form the union.

5.1 - Example 4: If $\mathbf{T} = \{a, e, i, o, u\}$, $\mathbf{V} = \{a, e, t, v\}$, find $\mathbf{T}\cup\mathbf{V}\cup\mathbf{T}$.

Solution:

$\mathbf{T}\cup\mathbf{V}\cup\mathbf{T} = \{a, e, i, o, u, t, v\}$. The union **T** with itself is equal to **T** since each element of **T** can only appear once.

5.1 - Example 5: If **K** = {a, b, c, d}, **L** = {a, d}, find **K**∪**L**.

Solution:

K∪**L** = **K** = {a,b,c,d}**.** All elements of **L** are also elements of **K**.

5.1 - Example 6: If **G** = {Billy, Mary, Harry}, find **G**∪ϕ.

Solution:

G∪ϕ = {Billy, Mary, Harry}∪ϕ = {Billy, Mary, Harry} = **G**

Solved Problems

5.1 - Solved Problem 1: If **G =** {cats, dogs, cattle, horses, sheep} and B = {horses, sheep, mules}, find **G**∪**B**.

Solution:

G∪**B** = {cats, dogs, cattle, horses, sheep, mules}. Remember, that duplication of elements is not allowed.

5.1 - Solved Problem 2: If **F =** {1, 3, 5, a, b} and **D** = {5, a, c, 0}, find **F**∪**D**.

Solution:

F∪**D** = {1,3,5,a,b,c,0}. Sets can consist of different types of elements.

5.1 - Solved Problem 3: If **K** = {a, t, mm}, **D** = {t, rd, m} and **R**= {1, 2, 3, 4, 5}, find **K**∪**D**∪**R**.

Solution:

K∪**D**∪**R=** {1, 2, 3, 4, 5, t, rd, m, a, mm}. Here we have a collection of all three sets.

5.1 - Solved Problem 4: If **T** = {7, 8, 9, 10}, **P** = {a, e, t, v}, find **T**∪**P**∪**T**.

Solution:

T∪**P**∪**T = T**∪**P** = {7, 8, 9, 10, a, e, t, v}. Again repetition is not allowed.

5.1 - Solved Problem 5: If **W** = {(H,H), (H,T), (T,H), (T,T)}, **X** = {(H,H), (T,T)}, find **W**∪**X** .

Solution:

W∪**X** = {(H,H), (H,T), (T,H), (T,T)}. **W** has 4 elements where each element is a pair of letters. For example the pair (H,H) is a single element.

5.1 - Solved Problem 6: Simplify **E** = {(1,2), (2,1), (3,3)}∪ϕ.

Solution:

E = {(1,2), (2,1), (3,3)}∪ϕ= **E** = {(1,2), (2,1), (3,3)}. The empty set has no elements. So the union is just the collection of elements of **E**.

Unsolved Problems with Answers.

5.1 - Problem 1: If **G** = {airplanes, automobiles, skates} and **D** = {horses, skates, tricycles, rockets}, find **G** ∪**D**.

Answer:

G ∪**D** = {airplanes, automobiles, skates, horses, tricycles, rockets}.

⇑ *Refer back to* **5.1 - Example 1 & 5.1 - Solved Problem 1**.

5.1 - Problem 2: If **F** = {California, Washington, a, b, Howard} and **D** = {1, 2, 3, 4, 5}, find **F** ∪**D**.

Answer:

F ∪**D** = {California, Washington, a, b, Howard, 1, 2, 3, 4, 5}.

⇑ *Refer back to* **5.1 - Example 2 & 5.1 - Solved Problem 2**.

5.1 - Problem 3: If **K** = {a, b, c}, **D** = {c, d, e} and **R**= {e, f, g}, find **K**∪**D**∪**R**.

Answer:

K ∪**D** ∪**R** = {a, b, c, d, e, f, g}.

⇑ *Refer back to* **5.1 - Example 3 & 5.1 - Solved Problem 3**.

5.1 - Problem 4: If **T** = {(a,b,c,d)}, **P** = {(a,b,d,c), (a,c,b,d), (a,d,c,b)}, find **T**∪**P**∪**T**.

Answer:

T∪**P**∪**T** = **T**∪**P** = {(a,b,c,d), (a,b,d,c), (a,c,b,d), (a,d,c,b)}.

⇑ *Refer back to* **5.1 - Example 4 & 5.1 - Solved Problem 4**.

5.1 - Problem 5: If **W** = {(1,1), (1,2) ,(2,1), (2,2)}, **X** = {(1,1), (2,2)}, find **W**∪**X.**

Answer:

W∪**X** = {(1,1), (1,2), (2,1), (2,2)}

⇑ *Refer back to* **5.1 - Example 5 & 5.1 - Solved Problem 5**.

5.1 - Problem 6: Simplify **E** = {1, 2, 3, 4,...}∪ϕ.

Answer:

E = {1, 2, 3, 4,...}∪ϕ = **E** = {1, 2, 3, 4,...}.

⇑ *Refer back to* **5.1 - Example 6 & 5.1 - Solved Problem 6**.

5.2 - What is the Intersection of Two or More Sets?

The intersection of two or more sets is the collection of all the distinct elements that are in common to these sets. This intersection results in a new set. The symbol for the intersection operator is ∩.

5.2 - Example 1: If **A** = {Mary, Jane, Harry, Bill, Howard} and **B** = {Jack, Bill, Frankie, Howard}, find **A**∩**B**.

Solution:

A∩**B =** {Bill, Howard}. The elements Bill and Howard are the only elements that are in both sets.

5.2 - Example 2: If A = {a, b, c}, **B** = {a, b, c, d, e}, **C** = {b, c, h, k}, find **B**∩**A**∩**C**.

Solution:

B∩**A**∩**C** = {b, c}. The elements b, c are the only elements that are in the three sets. The element a cannot be in the intersection of all three sets since a is only in the sets **A** and **B**.

5.2 - Example 3: If **E** = {a, c, e, f} and **A** = {a, f}, find **E**∩**A**.

Solution:

E∩**A** = {a, f} = **A**. All the elements of **A** are also elements of **E**.

5.2 - Example 4: If **P** = {7, 0, 5}, **E** = {1, 2, 3}, find **P**∩**E**.

Solution:

P∩**E** = ϕ. Since the sets **E** and **P** have no elements in common the intersection is empty.

5.2 - Example 5: If **R** = {1, 2, 3, 4, 5}, **I** = {1, 2, 3, 4}, **M** = {1, 3} find **R**∩**M**∩**I**.

Solution:

R∩**M**∩**I =** {1, 3}. Note that all the elements of **M** are in **R** and **I**. Also, all the elements of **I** are elements of **R**. Therefore, **R**∩**M**∩**I** = **M**.

5.2 - Example 6: If **A** = {(b,r), (r,b), (r,r), (b,b)}, **B** = {(r,b), (b,b)}, Find **A**∩**B**.

Solution:

A∩**B =** {(r,b), (b,b)}. All the elements of **B** are also elements of **A**. Therefore, **A**∩**B = B**.

Solved Problems

Solved Problem 1: If **A** = {M, J, H, B, K} and **B** = {J, B, F, H}, find **A**∩**B**.

Solution:

A ∩**B** = {J,B,H}**.** The intersection of sets is the set of all elements that are in common to all sets.

Solved Problem 2: If **A** = {M, J, H, B, K} and **B** = {J, B, F, H}, find **B**∩**A**∩**B.**

Solution:

B∩**A**∩**B** = {B, H, J}. Even though the set **B** is repeated twice, each element of **B** can only be represented once.

Solved Problem 3: If **E =** {1, 2, 3, 4, 5} and **D =** {4,6,8,10}, find **E**∩**D**.

Solution:

E∩**D =** {4}. The element 4 is the only element in both sets **E** and **D**.

Solved Problem 4: If **P** = {(h,h), (t,t)}, **E** = {h, t}, find **P**∩**E**.

Solution:

P∩**E = ϕ.** There are no elements in common to both sets. (h,h) is a different element then h.

Solved Problem 5: If **R** = {a, b, c, d, e, f}, **I** = {a, b, c, d}, **M** = {c, d}, find **R**∩M∩**I**.

Solution:

R∩**M**∩**I =** {c, d} = **M**. All elements of **I**, **M** are also elements of **R**.

Solved Problem 6: If **T** = {(r,r,b), (r,b,r), (b,r,r)}, **V** = {(r,b,r), (r,r,r)},**R** = {(r,b,r)} find **T**∩**V**∩**R**.

Solution:

T∩**V**∩**R** = {(r,b,r)}. The element (r,b,r) is the only element that is in all three sets.

Unsolved Problems with Answers

5.2 - Problem 1: If **A** = {New York, Washington D.C., Miami, Berkeley} and **B** = {New York, Miami, Las Vegas}, find **A∩B**.

Answer:

A∩B = {Miami, New York}.

⇑ *Refer back to* **5.2 - Example 1 & 5.2 - Solved Problem 1.**

5.2 - Problem 2: If **A** = {(1,2,3), (1,2,6), (1,3,5)} and **B** = {(1,2,3), (1,1,1), (1,3,5)}, find **B∩A∩B**.

Answer:

B∩A∩B = {(1,2,3), (1,3,5)}.

⇑ *Refer back to* **5.2 - Example 2 & 5.2 - Solved Problem 2.**

5.2 - Problem 3: If **E =** {{1, 2, 3}, {2, 4, 5}} and **D =** {{3, 4, 5}, {1, 2, 3}, {1}}, find **E∩D**.

Answer:

E∩D = {{1, 2, 3}}.

⇑ *Refer back to* **5.2 - Example 3 & 5.2 - Solved Problem 3**.

5.2 - Problem 4: If **P** = {Mary Smith, Howard Jones}, **E**={Mary, Smith, Howard, Jones}, find **P∩E**.

Answer:

P∩E = ϕ

⇑ *Refer back to* **5.2 - Example 4 & 5.2 - Solved Problem 4**.

5.2 - Problem 5: If **R** = {1, 2, 3, 4, 5,...}, **I** = {2, 3, 4,**...**}, **M** = {4, 5, 6,**...**}, find **R∩M∩I**.

Answer:

R∩M∩I = {4, 5, 6,...}.

⇑ *Refer back to* **5.2 - Example 5 & 5.1 - Solved Problem 5**.

5.2 - Problem 6: If **T** = {(1,1), (1,2), (1,3), (1,4), (1,5), (1,6)}, **S** = {(1,1), (2,1), (3,1), (4,1), (5,1), (6,1)}, find **S∩T**.

Answer:

S∩T = {(1, 1)}

⇑ *Refer back to* **5.2 - Example 6 & 5.1 - Solved Problem 6**.

Before, explaining the complement operator, we need to define and explain when a set is a subset or a proper subset of another set. Also, we need to define the idea of a universal set.

5.3 - What is a Subset?

A set **C** is a subset of a set **D** if all the elements of **C** are also members of **D**. The symbol for subset is ⊆. If **C** is a subset of **D** we write **C** ⊆ **D**.

5.3 - Example 1: If **F** = {1, 4, 6} and **B** = {8, 11, 4, 6, 1} is **F** ⊆ **B**?

Solution:

Yes. All the elements of **F** are also elements of **B**.

5.3 - Example 2: Write out all subsets of **H** = {1, 4}.

Solution:

{1, 4} ⊆ {1, 4}

{1} ⊆ {1, 4}

{4} ⊆ {1, 4}

ϕ ⊆ {1, 4}

Therefore, all subsets are {1, 4}, {1}, {4}, ϕ.

Note: It can be shown that the empty set is a subset of all sets (See supplementary problem 13).

5.3 - Example 3: Write out all the subsets of **H** = {J, K, L}.

Solution:

{J} ⊆ {J, K, L}

{K}⊆ {J, K, L}

{L}⊆ {J, K, L}

{J, K}⊆ {J, K, L}

{J, L}⊆ {J, K, L}

{K, L}⊆ {J, K, L}

{J, K, L}⊆ {J, K, L}

ϕ ⊆ {J, K, L}

Therefore, all subsets are {J},{K},{L},{J, K},{J, L},{K, L},{J, K, L}, ϕ.

Note: We always assume that the empty set is a subset of all sets.

5.3 - Example 4: Write out all subsets of **W =** {(r,r), (b,b)}.

Solution:

{(r,r)} ⊆ {(r,r), (b,b)}

{(b,b)} ⊆ {(r,r), (b,b)}

{(r,r),(b,b)} ⊆ {(r,r), (b,b)}

ϕ ⊆ {(r,r), (b,b)}

Therefore, all subsets are {(r,r)}, {(b,b)}, {(r,r),(b,b)}, ϕ.

Solved Problems

5.3 - Solved Problem 1: Is **F** = {a, b, c, d,} ⊆ **B**= {0, 1, a, b, 3, c, d}?

Solution: Yes. All elements of **F** are also elements of **B**.

5.3 - Solved Problem 2: Find all subsets of **U** = {a, b}.

Solution:

ϕ ⊆ {a, b}

{a} ⊆ {a, b}

{b} ⊆ {a, b}

{a, b} ⊆ {a ,b}

Therefore, ϕ,{a},{b},{a, b} are all subsets of {a, b}.

5.3 - Solved Problem 3: Find all subsets of **U** = {a, b, c}.

Solution:

ϕ ⊆ {a, b, c}

$\{a\} \subseteq \{a, b, c\}$
$\{b\} \subseteq \{a, b, c\}$
$\{c\} \subseteq \{a, b, c\}$
$\{a, b\} \subseteq \{a, b, c\}$
$\{a, c\} \subseteq \{a, b, c\}$
$\{b, c\} \subseteq \{a, b, c\}$
$\{a, b, c\} \subseteq \{a, b, c\}$

Therefore, ϕ, {a}, {b}, {c}, {a, b}, {a, c}, {b, c}, {a, b, c} are the subsets of {a, b, c}.

5.3 - Solved Problem 4: Write out all subsets of **E** = {(r,r,b), (r,b,r), (b,r,r)}.

Solution:

$\{(r,r,b)\} \subseteq \{(r,r,b), (r,b,r), (b,r,r)\}$

$\{(r,b,r)\} \subseteq \{(r,r,b), (r,b,r), (b,r,r)\}$

$\{(b,r,r)\} \subseteq \{(r,r,b), (r,b,r), (b,r,r)\}$

$\{(b,r,r), (r,b,r)\} \subseteq \{(r,r,b), (r,b,r), (b,r,r)\}$

$\{(b,r,r), (r,r,b)\} \subseteq \{(r,r,b), (r,b,r), (b,r,r)\}$

$\{(r,r,b), (r,b,r)\} \subseteq \{(r,r,b), (r,b,r), (b,r,r)\}$

$\{(r,r,b), (r,b,r), (b,r,r)\} \subseteq \{(r,r,b), (r,b,r), (b,r,r)\}$

$\phi \subseteq \{(r,r,b), (r,b,r), (b,r,r)\}$

Therefore, the following are subsets of **E**:

ϕ, {(r,r,b)}, {(r,b,r)}, {(b,r,r)}, {(b,r,r),(r,b,r)}, {(b,r,r),(r,r,b)},{(r,r,b), (r,b,r)}, {(r,r,b), (r,b,r), (b,r,r)}.

Unsolved problems with answers

5.3 - Problem 1: Is **F** = {a, b, c, d, e} ⊆ **B** = {a, b, c, d}?

Answer:

No.

⇑ *Refer back to* **5.3 - Example 1& 5.3 - Solved Problem 1**.

5.3 - Problem 2: Find all subsets of **U** = {{a, b}, {a, c}}

Answer:

ϕ, {{a,b}}, {{a,c}}, {{a,b},{a,c}}

⥣ *Refer back to* **5.3 - Example 2 & 5.3 - Solved Problem 2**.

5.3 - Problem 3: Find all subsets of **U** = {1, 2, 3}.

Answer:

ϕ, {1}, {2}, {3}, {1,2}, {1,3}, {2,3}, {1,2,3}

⥣ *Refer back to* **5.3 - Example 3 & 5.3 - Solved Problem 3**.

5.3 - Problem 4: Write out all subsets of **E** = {(r,r,b), (r,b,r), (b,r,r), (r,r,r)}.

Answer:

ϕ,

{(r,r,b)}, {(r,b,r)}, {(b,r,r)}, {(r,r,r)},

{(b,r,r), (r,b,r)}, {(b,r,r), (r,r,b)}, {(r,r,b), (r,b,r)},

{(r,r,r), (r,b,r)}, {(r,r,r), (r,r,b)}, {(r,r,r), (b,r,r)},

{(b,r,r), (r,b,r), (r,r,r)}, {(b,r,r), (r,r,b), (r,r,r)},

{(r,r,b), (r,b,r), (r,r,r)}, {(r,r,b), (r,b,r), (b,r,r)}, {(r,r,b), (r,b,r), (b,r,r), (r,r,r)}

⥣ *Refer back to* **5.3 - Example 4 & 5.3 - Solved Problem 4**.

5.4 - What is a Proper Subset?

A set **C** is a proper subset of a set **D** if all the elements of **C** are also members of **D** and there is at least one element of **D** that is not in **C**. The symbol for the proper subset is ⊂. If **C** is a proper subset of **D** then we indicate this by **C** ⊂ **D**.

Note: If **A** is a proper subset of B then **A** is also a subset of **B**.

5.4 - Example 1: If **F** = {1, 4, 6} and **B** = {8, 11, 4, 6, 1} is **F** ⊂ **B**?

Solution:

Step 1: The elements of **F** are 1,4,6 which are also elements of **B.** Therefore, **F** is a subset of **B**.

Step 2: The set **B** has elements 8,11 that are not in **F**.

Therefore, **F** is a proper subset of **B**: **F** ⊂ **B**.

5.4 - Example 2: Find all proper subsets of **W** = {1, 3, 5}.

Solution:

{1} ⊂ {1 ,3 ,5}
{3} ⊂ {1, 3, 5}
{5} ⊂ {1, 3, 5}
{1, 3} ⊂ {1, 3, 5}
{1, 5} ⊂ {1,3,5}
{3, 5} ⊂ {1, 3, 5}
ϕ ⊂ {1, 3, 5}
Therefore, {1}, {3}, {5}, {1, 3}, {1, 5}, {3, 5}, ϕ are proper subsets of **W**.

5.4 - Example 3: Find all proper subsets of **W** = {(h,h), (t,t)}.

Solution:

{(h,h)} ⊂ {(h,h), (t,t)}

{(t,t)} ⊂ {(h,h), (t,t)}

ϕ ⊂ {(h,h), (t,t)}

Therefore, {(h,h)}, {(t,t)}, ϕ are the proper subsets of **W**.

Solved Problems

5.4 - Solved Problem 1: If **F** = {1, 2, 3} and **B** = {0, 1, 2, 3,4, 5} is **F** ⊂ **B** ?

Solution:

Step 1: The elements of **F** are 1,2,3 which are also elements of **B.** Therefore, **F** is a subset of **B**.

Step 2: The set **B** has elements 0,4,5 that are not in **F**.

Therefore, **F** is a proper subset of **B**: **F** ⊂ **B**.

5.4 - Solved Problem 2: Find all proper subsets of **W** = {a, b, z}.

Solution:

ϕ ⊂ {a, b, z}
{a} ⊂ {a ,b, z}
{b} ⊂ {a ,b, z}
{z} ⊂ {a, b, z}
{a, b} ⊂ {a, b, z}
{a, z} ⊂ {a, b, z}
{b, z} ⊂ {a, b, z}

Therefore, the proper subsets are ϕ, {a}, {b}, {z}, {a, b}, {a, z}, {b, z}.

Each of these subsets lack at least one element of **W**.

5.4 - Solved Problem 3: Find all proper subsets of **W** = {(h,h,h), (t,t,t)}.

Solution:

ϕ ⊂ {(h,h,h), (t,t,t)}
{(h,h,h)} ⊂ {(h,h,h), (t,t,t)}
{(t,t,t)} ⊂ {(h,h,h), (t,t,t)}

Therefore, all proper subsets of **W** are ϕ, {(h,h,h)}, {(t,t,t)}.

Unsolved Problems with Answers.

5.4 - Problem 1: If **F** = {8, 1, 2, 3} and **B** = {0, 1, 2, 3, 4, 5} is **F** ⊂ **B** ?

Answer:

no.

⇑ *Refer back to* **5.4 - Example 1 & 5.4 - Solved Problem 1**.

5.4 - Problem 2: Find all proper subsets of **W** = {Billy, Sally, Jill}.

Answer:

ϕ, {Billy}, {Sally}, {Jill}, {Billy,Sally}, {Billy,Jill}, {Sally,Jill}

⇑ *Refer back to* **5.4 - Example 2 & 5.3 - Solved Problem 2**.

Problem 3: Find all proper subsets of **W** = {(1,1,1), (2,2,2), (1,2,1)}.

Answer:

ϕ,{(1,1,1)}, {(2,2,2)}, {(1,2,1)}, {(1,1,1), (2,2,2)}, {(1,1,1), (1,2,1)}, {(2,2,2), (1,2,1)}

⇑ *Refer back to* **5.4 - Example 3 & 5.4 - Solved Problem 3** .

5.5 - What is a Universal Set?

> Assume **A,B,C.** etc are sets that are subsets of the same set indicated by $\mathcal{U}$.
>
> The set $\mathcal{U}$ is called the universal set.

5.5 - Example 1: Assume **A** = {1, 3, 5, 7, 9}, **B** = {1, 6, 8, 2}. Is **C =** {1, 2, 3, 4, 5, 6, 7, 8, 9, 10} a universal set?

Solution:

Since {1, 3, 5, 7, 9} ⊂ {1, 2, 3, 4, 5, 6, 7, 8, 9, 10} and {1, 6, 8, 2} ⊂ {1, 2, 3, 4, 5, 6, 7, 8, 9, 10} then {1, 2, 3, 4, 5, 6, 7, 8, 9, 10} = $\mathcal{U}$ is a universal set.

5.5 - Example 2: Assume **A** = {b, c, d}, **F** = {a, b, d, e, g}. Is {a, b, c, d, e, f} a universal set?

Solution: Since **F** contains the element g but g is not a member of {a, b, c, d, e, f} then {a, b, c, d, e, f} is not a universal set.

5.5 - Example 3: Find the smallest universal set of **Q** = {1, 2, 3, 4, 5}**,** and **P** = {0, 1, 2, 3}.

Solution:

The smallest universal set containing the sets **Q** and **P** can be formed by taking the union of these two sets:

Q∪**P** ={1, 2, 3, 4, 5}∪{0, 1, 2, 3} = {0, 1, 2, 3, 4, 5}.

Therefore, $\mathcal{U}$ = **Q**∪**P** = {0, 1, 2, 3, 4, 5} is the smallest universal set that only contains the elements in the sets **Q** and **P**.

5.5 - Example 4: Find the smallest universal set of

Q = {(w,w,l), (w,l,w), (l,w,w,)}**,**

P = {(l,l,w), (l,w,l), (w,l,l)},

E = {(w,w,w), (l,l,l)}.

Solution:

The smallest universal set containing the sets **Q** and **P** can be formed by taking the union of these three sets:

Q∪**P**∪**E** = {(w,w,l), (w,l,w), (l,w,w,)}∪{(l,l,w), (l,w,l), (w,l,l)}∪{(w,w,w), (l,l,l)}

Therefore, 𝒰 = **Q**∪**P**∪**E** = {(w,w,l), (w,l,w), (l,w,w,), (l,l,w), (l,w,l), (w,l,l), (w,w,w), (l,l,l)}.

Solved Problems.

5.5 - Solved Problem 1: Assume **A** = {e, f, g,...,z}, **B** = {w, x, y, z}. Is {a, b, c, d,..., z} a universal set?

Solution:

Since {e, f, g,..., z} ⊂ {a, b, c, d,..., z} and {w, x, y, z}⊂ {a, b, c, d,..., z}, the set 𝒰 = {a, b, c, d,..., z} is a universal set of these two sets.

5.5 - Solved Problem 2: Assume **A** = {7, 8, 9}, **F** = {0, 1, 2, 3}. Is {1, 2, 3, 4, 5, 6, 7, 8, 10} a universal set?

Solution:

Since 0 is an element of **F** but not a member of {1, 2, 3, 4, 5, 6, 7, 8, 10}, the set {1, 2, 3, 4, 5, 6, 7, 8, 10} is not a universal set of these two sets.

5.5 - Solved Problem 3: Find the smallest universal set of **Q =** {a, b, c}**,** and **P** = {1, 2}.

Solution:

The union of **P** and **Q** is the smallest universal set:

Q∪**P** = {a, b, c}∪{1, 2} = {a, b, c, 1,2} = 𝒰 .

5.5 - Solved Problem 4: Find the smallest universal set of

Q = {(r,r,g), (g,r,r), (r,g,r)}**,**

P = {(g,g,r), (g,r,g), (r,g,g)},

E = {(b,b,r), (b,r,b), (r,b,b)}.

Solution:

The union of **Q,P,E** is the smallest universal set:

$\mathfrak{U}$ = **Q**∪**P**∪**E** = {(r,r,g), (g,r,r), (r,g,r)}∪{(g,g,r), (g,r,g), (r,g,g)}∪ {(b,b,r), (b,r,b), (r,b,b)} =

{(r,r,g), (g,r,r), (r,g,r), (g,g,r), (g,r,g), (r,g,g), (b,b,r), (b,r,b), (r,b,b)}.

Unsolved problems with Answers

5.5 - Problem 1: Assume **A** = {horses}, **B** = {cattle, pigs}. Is {horses, cattle, pigs} a universal set?

Answer:

Yes.

⇑ *Refer back to* **5.5 - Example 1 & 5.5 - Solved Problem 1**.

5.5 - Problem 2: Assume **A** = {horses}, **B** = {cats, cattle, pigs}. Is {horses, cattle, pigs} a universal set?

Answer:

no.

⇑ *Refer back to* **5.5 - Example 2 & 5.5 - Solved Problem 2**.

5.5 - Problem 3: Find the smallest universal set of **Q** = {(h,h), (h,t), (t,h), (t,t)}, and **P** = {h, t}.

Answer:

$\mathfrak{U}$ = {(h,h), (h,t), (t,h), (t,t), h, t}

⇑ *Refer back to* **5.5 - Example 3 & 5.5 - Solved Problem 3**.

5.5 - Problem 4: Find the smallest universal set of

Q = {(h,h), (h,t), (t,h), (t,t)}**,**

P = {(h,h), (t,t)},

E = {(t,h), (h,t)}.

Answer:

$\mathfrak{U}$ = {(h,h), (h,t), (t,h), (t,t)}

⇑ *Refer back to* **5.5 - Example 4 & 5.5 - Solved Problem 4**.

5.6 - What is the Complement of a Set?

Assume **D** is a subset of the universal set $\mathcal{U}$ (**D** ⊆ $\mathcal{U}$). The complement of **D** is the set consisting of all elements in $\mathcal{U}$ not contained in **D** . The complement of **D** is noted by **D**′.

5.6 - Example 1: Assume $\mathcal{U}$ = {a, b, c, d, e, f}, and **A** = {b, c, d}. Find **A**′.

Solution:

Step 1: **A** = {b, c, d} ⊂ {a, b, c, d, e, f}

Step 2: **A**′ are the elements of {a, b, c, d, e, f} that are not in **A:** {a,e,f}.

Step 3: Therefore, **A**′ **=** {a, e, f}.

5.6 - Example 2: Assume $\mathcal{U}$ = {(h,h), (t,t,), (h,t), (t,h)} and **C** ={(h,t), (t,h)}. Find **C**′.

Solution:

Step 1: **C** = {(h,t), (t,h)} ⊂ {(h,h), (t,t,), (h,t), (t,h)}

Step 2: **C**′ are the elements of {(h,h), (t,t,), (h,t), (t,h)} that are not in **C**: {(h,h), (t,t)}

Step 3: Therefore, **C**′ = {(h,h), (t,t)}.

Solved Problems

5.6 - Solved Problem 1: Assume $\mathcal{U}$ = {Harry, Mary, Jane, Bill, Frankie, Howard, Seth} and **K** = {Harry, Mary, Jane, Bill, Frankie}. Find **K**′.

Solution:

Step 1: **K** = {Harry, Mary, Jane, Bill, Frankie} ⊂ {Harry, Mary, Jane, Bill, Frankie, Howard, Seth}

Step 2: **K**′ are the elements of {Harry, Mary, Jane, Bill, Frankie, Howard, Seth} that are not in **K**: {Howard, Seth}.

Step 3: Therefore, **K**′ = {Howard, Seth}.

5.6 - Solved Problem 2: If $\mathcal{U}$ = {(h,h,h), (t,t,t), (h,h,t), (h,t,h), (t,h,h), (t,t,h), (t,h,t), (h,t,t)} and **A =** {(h,h,h), (t,t,t)}, find **A**′.

Solution:

Step 1: **A** = {(h,h,h), (t,t,t)} ⊂ {(h,h,h), (t,t,t), (h,h,t), (h,t,h), (t,h,h), (t,t,h), (t,h,t), (h,t,t)}

Step 2: **A**′ are the elements of {(h,h,h), (t,t,t), (h,h,t), (h,t,h), (t,h,h), (t,t,h), (t,h,t), (h,t,t)}

that are not in **A**: {(t,t,h), (t,h,t), (h,t,t), (h,h,t), (h,t,h), (t,h,h)}.

Step 3: Therefore, **A**′ = {(t,t,h), (t,h,t), (h,t,t), (h,h,t), (h,t,h), (t,h,h)}.

Unsolved Problems with Answers

5.6 - Problem 1: If 𝔘 = {a, b, c, d} and **D** = {a, d} Find **D**′.

Answer:

D′ = {b, c}

⇑ *Refer back to* **5.6 - Example 1 & 5.6 - Solved Problem 1**.

5.6 - Problem 2: If 𝔘 = {(h,h,h), (t,t,t), (h,h,t), (h,t,h), (t,h,h), (t,t,h), (t,h,t), (h,t,t)} and
A = {(h,h,t), (h,t,h), (t,h,h)} find **A**′.

Answer:

A′ = {(h,h,h), (t,t,t), (t,t,h), (t,h,t), (h,t,t)}

⇑ *Refer back to* **5.6 - Example 2 & 5.6 - Solved Problem 2** .

5.7 - Combining operators.

The three set operators can be combined using parentheses ( ) to determine the order of operations. The rule is to evaluate the set operations inside the parentheses first.

For the following examples, assume 𝔘 = {1, 2, 3, 4, 5, 6, 7, 8, 9, 10}, A = {1, 3, 5, 7, 9}, **B** = {7, 8, 9, 10}, and **D** = {2, 4, 6, 8, 10}.

5.7 - Example 1: Find (**A**∪**B**)′.

Solution:

Step 1: First find the union:

A∪**B** = {1, 3, 5, 7, 9}∪{7, 8, 9, 10} = {1, 3, 5, 7, 8, 9, 10}.

Step 2: Next take the complement of the event in step 1:

$(\mathbf{A}\cup\mathbf{B})' = \{1, 3, 5, 7, 8, 9, 10\}' = \{2, 4, 6\}$.

5.7 - Example 2: Find $(\mathbf{A}\cap \mathbf{B})'$.

Solution:

Step 1: First find the intersection:

$\mathbf{A}\cap\mathbf{B} = \{1, 3, 5, 7, 9\}\cap\{7, 8, 9, 10\} = \{7, 9\}$.

Step 2: Next find the complement of the event in step 1:

$(\mathbf{A}\cap\mathbf{B})' = \{7, 9\}' = \{1, 2, 3, 4, 5, 6, 8, 10\}$.

5.7 - Example 3: Find $(\mathbf{A}\cup\mathbf{B})\cap\mathbf{D}$.

Solution:

Step 1: First find the union of **A** and **B:** $\mathbf{A}\cup\mathbf{B} = \{1, 3, 5, 7, 9\}\cup\{7, 8, 9, 10\} = \{1, 3, 5, 7, 9, 8, 10\}$

Step 2: Next find the intersection of **D** with the event in step 1:

$(\mathbf{A}\cup\mathbf{B})\cap\mathbf{D} = \{1, 3, 5, 7, 9, 8, 10\}\cap\{2, 4, 6, 8, 10\} = \{8, 10\}$

5.7 - Example 4: Find $(\mathbf{A}\cap\mathbf{B})\cup\mathbf{D}$.

Solution:

Step 1: First find the intersection of **A** and **B**:

$\mathbf{A}\cap\mathbf{B} = \{1, 3, 5, 7, 9\}\cap\{7, 8, 9, 10\} = \{7, 9\}$.

Step 2: Next find the union of **D** with the event in step 1:

$(\mathbf{A}\cap\mathbf{B})\cup\mathbf{D} = \{7, 9\}\cup\{2, 4, 6, 8, 10\} = \{2, 4, 6, 8, 10, 7, 9\}$.

5.7 - Example 5: Find $(\mathbf{A}\cup\mathbf{B})'\cap\mathbf{D}$.

Solution:

Step 1: First find the union of **A** and **B**:

$\mathbf{A}\cup\mathbf{B} = \{1, 3, 5, 7, 9\}\cup\{7, 8, 9, 10\} = \{1, 3, 5, 7, 8, 9, 10\}$.

Step 2: Next find the complement of the event in step 1:

$(\mathbf{A} \cup \mathbf{B})' = \{1, 3, 5, 7, 8, 9, 10\}' = \{2, 4, 6\}$

Step 3: Find the intersect of **D** with the event above:

$(\mathbf{A} \cup \mathbf{B})' \cap \mathbf{D} = \{2, 4, 6\} \cap \{2, 4, 6, 8, 10\} = \{2, 4, 6\}$

5.7 - Example 6: Find $(\mathbf{A} \cup \mathbf{B}) \cap \mathbf{D}'$.

Solution:

Step 1: First find the union of **A** and **B**:

$\mathbf{A} \cup \mathbf{B} = \{1, 3, 5, 7, 9\} \cup \{7, 8, 9, 10\} = \{1, 3, 5, 7, 8, 9, 10\}$

Step 2: Find the complement of **D**:

$\mathbf{D}' = \{1, 3, 5, 7, 9\}$

Step 3: Find the intersection of the event in step 2 with the event in step 1:

$(\mathbf{A} \cup \mathbf{B}) \cap \mathbf{D}' = \{1, 3, 5, 7, 8, 9, 10\} \cap \{1, 3, 5, 7, 9\} = \{1, 3, 5, 7, 9\}$

Solved Problems

For the following examples, assume $\mathfrak{U} = \{a, e, i, o, u\}$, $A = \{a, e, i\}$,$\mathbf{B} = \{a, o\}$, and $\mathbf{D} = \{i, u\}$.

5.7 - Solved Problem 1: Find $(\mathbf{A} \cup \mathbf{B})'$.

Solution:

Step 1: $\mathbf{A} \cup \mathbf{B} = \{a, e, i\} \cup \{a, o\} = \{a, e, i, o\}$

Step 2: $(\mathbf{A} \cup \mathbf{B})' = \{a, e, i, o\}' = \{u\}$

Step 3: Therefore, $(\mathbf{A} \cup \mathbf{B})' = \{u\}$.

5.7 - Solved Problem 2: Find $(\mathbf{A} \cap \mathbf{B})'$.

Solution:

Step 1: $\mathbf{A} \cap \mathbf{B} = \{a, e, i\} \cap \{a, o\} = \{a\}$

Step 2: $(\mathbf{A} \cap \mathbf{B})' = \{a\}' = \{e, i, o, u\}$

Step 3: Therefore, $(\mathbf{A} \cap \mathbf{B})' = \{e, i, o, u\}$.

5.7 - Solved Problem 3: Find (**A**∪**B**)∩**D**.

Solution:

Step 1: (**A**∪**B**) = {a, e, i}∪{a, o} = {a, e, i, o}

Step 2: (**A**∪**B**)∩**D** ={a, e, i, o}∩{i, u} = {i}

Step 3: Therefore, (**A**∪**B**)∩**D** = {i}

5.7 - Solved Problem 4: Find (**A**∩**B**)∪**D**.

Solution:

Step 1: (**A**∩**B**) **=** {a, e, i}∩{a, o} = {a}.

Step 2: (**A**∩**B**)∪**D** **=** {a}∪{i, u} = {a, i, u}

Step 3: Therefore, (**A**∩**B**)∪**D** **=** {a, i, u}.

5.7 - Solved Problem 5: Find (**A**∪**B**)′∩**D**.

Solution:

(**A**∪**B**)′∩**D** **=** {a, e, i, o}′∩{i, u} = {u}∩{i, u} = {u}

Step 1: **A**∪**B** = {a, e, i}∪{a, o} = {a, e, i, o}

Step 2: (**A**∪**B**)′= {a, e, i, o}′ = {u}

Step 3: (**A**∪**B**)′∩**D** **=** {u}∩{i, u} = {u}

Step 4: Therefore,(**A**∪**B**)′∩**D** = {u}

5.7 - Solved Problem 6:Find (**A**∪**B**)∩**D**′.

Solution:

(**A**∪**B**)∩**D**′ = {a, e, i, o}∩{a, e, o} = {a, e, o}

Step 1: **A**∪**B** = {a, e, i}∪{a, o} = {a, e, i, o}

Step 2: **D**′ = {i, u}′ = {a, e, o}

Step 3: (**A**∪**B**)∩**D**′ = {a, e, i, o}∩{a, e, o} = {a, e, o}

Step 4: Therefore,(**A**∪**B**)∩**D**′ = {a, e, o}.

Unsolved Problems with Answers

For the following examples, assume $\mathcal{U}$ = {1, 2, 3, 4, 5,..., 100}**, A =** {2, 4, 6, 8, 10,..., 100}, **B** = {1**,** 3, 5, 7, 9, **... ,** 99}, and **D** = {1, 2, 3, 4, 5}.

5.7 - Problems 1: Find (**D**∪**B**)′.

Answer:

{6, 8, 10, 12,..., 100}

⇑ *Refer back to* **5.7 - Example 1 & 5.7 - Solved Problem 1**.

5.7 - Problem 2: Find (**D**∩**B**)′.

Answer:

{2, 4, 6, 7, 8, 9,..., 100}

⇑ *Refer back to* **5.7 - Example 2 & 5.7 - Solved Problem 2**.

5.7 - Problem 3: Find (**A**∪**B**)∩**D**.

Answer:

{1, 2, 3, 4, 5}

⇑ *Refer back to* **5.7 - Example 3 & 5.7 - Solved Problem 3**.

5.7 - Problem 4: Find (**A**∩**B**)∪**D**.

Answer:

{1, 2, 3, 4, 5}

⇑ *Refer back to* **5.7 - Example 4 & 5.7 - Solved Problem 4**.

5.7 - Problem 5: Find (**A**∪**D**)′∩**B**.

Answer:

{7, 9, 11,.., 99}

⇑ *Refer back to* **5.7 - Example 5 & 5.7 - Solved Problem 5**.

5.7 - Problem 6: Find $(\mathbf{A}\cup\mathbf{B})\cap\mathbf{D}'$.

Answer:

{6, 7, 8, 9,..., 100}

⇑ *Refer back to* **5.7 - Example 6 & 5.7 - Solved Problem 6**.

5.8 - DeMorgan Laws

The following laws of DeMorgan are very useful when combining sets:

$(\mathbf{A}\cup\mathbf{B})' = \mathbf{A}'\cap\mathbf{B}'$

$(\mathbf{A}\cap\mathbf{B})' = \mathbf{A}'\cup\mathbf{B}'$

5.8 - Example 1: Assume $\mathcal{U}$ = {1, 2, 3, 4, 5,..., 10}**,** **A** = {4, 5, 6, 7, 8}, and **B** = {1, 2, 4, 5, 9}. Show

(a). $(\mathbf{A}\cup\mathbf{B})' = \mathbf{A}'\cap\mathbf{B}'$.

(b). $(\mathbf{A}\cap\mathbf{B})' = \mathbf{A}'\cup\mathbf{B}'$.

Solutions:

➤ **(a)**.
Left Side:

Step 1: $\mathbf{A}\cup\mathbf{B}$ = {4, 5, 6, 7, 8}∪{1, 2, 4, 5, 9} = {1, 2, 4, 5, 6, 7, 8, 9}

Step 2: $(\mathbf{A}\cup\mathbf{B})'$ = {1, 2, 4, 5, 6, 7, 8, 9}′ = {3, 10}

Right Side:

Step 1: $\mathbf{A}'$ = {4, 5, 6, 7, 8}′ = {1, 2, 3, 9, 10}

Step 2: $\mathbf{B}'$ = {1, 2, 4, 5, 9}′ = {3, 6, 7, 8, 10}

Step 3: $\mathbf{A}'\cap\mathbf{B}'$ = {3, 10}

Step 4: Therefore, $(\mathbf{A}\cup\mathbf{B})' = \mathbf{A}'\cap\mathbf{B}'$.

➤ **(b)**.
Left Side:

Step 1: $\mathbf{A}\cap\mathbf{B} = \{4, 5, 6, 7, 8\}\cap\{1, 2, 4, 5, 9\} = \{4, 5\}$

Step 2: $(\mathbf{A}\cap\mathbf{B})' = \{4, 5\}' = \{1, 2, 3, 6, 7, 8, 9, 10\}$

Right Side:

Step 1: $\mathbf{A}' = \{4, 5, 6, 7, 8\}' = \{1, 2, 3, 9, 10\}$

Step 2: $\mathbf{B}' = \{1, 2, 4, 5, 9\}' = \{3, 6, 7, 8, 10\}$

Step 3: $\mathbf{A}'\cup\mathbf{B}' = \{1, 2, 3, 9, 10\}\cup\{3, 6, 7, 8, 10\} = \{1, 2, 3, 6, 7, 8, 9, 10\}$

Step 4: Therefore, $(\mathbf{A}\cap\mathbf{B})' = \mathbf{A}'\cup\mathbf{B}'$.

Solved Problems

5.8 - Solved Problem 1: Assume $\mathfrak{U} = \{a, e, i, o, u\}$, $\mathbf{A} = \{a, e, i\}$, and $\mathbf{B} = \{a, o\}$. Show

(a). $(\mathbf{A}\cup\mathbf{B})' = \mathbf{A}'\cap\mathbf{B}'$.

(b). $(\mathbf{A}\cap\mathbf{B})' = \mathbf{A}'\cup\mathbf{B}'$.

Solutions:

➤ **(a)**.
Left Side:

Step 1: $\mathbf{A}\cup\mathbf{B} = \{a, e, i\}\cup\{a, o\} = \{a, e, i, o\}$

Step 2: $(\mathbf{A}\cup\mathbf{B})' = \{a, e, i, o\}' = \{u\}$

Right Side:

Step 1: $\mathbf{A}' = \{a, e, i\}' = \{o, u\}$

Step 2: $\mathbf{B}' = \{a, o\}' = \{e, i, u\}$

Step 3: $\mathbf{A}'\cap\mathbf{B}' = \{u\}$

Step 4: Therefore, $(\mathbf{A}\cup\mathbf{B})' = \mathbf{A}'\cap\mathbf{B}'$.

➤ **(b)**.
Left Side:

Step 1: $(\mathbf{A}\cap\mathbf{B}) = [\{a, e, i\}\cap\{a, o\}] = \{a\}$

Step 2: (**A**∩**B**)′ = [{a, e, i}∩{a, o}]′ = {a}′ = {e, i, o, u}

Right Side:

Step 1: **A**′ = {a, e, i}′ = {o, u}

Step 2: **B**′ = {a, o}′ = {e, i, u}

Step 3: **A**′∪**B**′ = {o, u}∪{e, i, u} = {e, i, o, u}

Step 4: Therefore, (**A**∩**B**)′ = **A**′∪**B**′.

Unsolved Problems with Answers

5.8 - Problem 1: Assume 𝔘 = {(h,h), (h,t), (t,h), (t,t)}, **A =** {(h,h), (h,t), (t,h)}, and **B** = {(h,t), (t,h)}. Show

(a). (**A**∪**B**)′ = **A**′∩**B**′.

(b). (**A**∩**B**)′ = **A**′∪**B**′.

Answers:

➤ **(a)**.
Left Side:

(**A**∪**B**)′ **=** {(t,t)}

Right Side:

A′∩**B**′ = {(t,t)}

➤ **(b)**.
Left Side:

(**A**∩**B**)′ = {(h,h), (t,t)}

Right Side:

A′∪**B**′ **=** {(h,h), (t,t)}

⇑ *Refer back to* **5.8 - Example 1 & 5.8 - Solved Problem 1**.

Supplementary Problems

For the following questions, assume arbitrary sets.

1. If $\mathbf{A} \subseteq \mathbf{B}$ and $\mathbf{B} \subseteq A$, show that $\mathbf{A} = \mathbf{B}$.

For questions 2 - 11, perform the indicated operations and simplify.

2. $\mathbf{D}\cap\mathbf{D}$ 3. $\mathbf{F}\cap\mathbf{F}'$ 4. $\mathbf{F}\cup\mathbf{F}'$ 5. ϕ' 6. $\phi\cap\mathbf{G}$ 7. $\phi\cup\mathbf{G}$ 8. $\mathfrak{U}\cup\mathbf{G}$ 9. $\mathfrak{U}\cap\mathbf{G}$ 10. $\mathfrak{U}'$ 11. $\mathbf{F}''$

12. If $\mathbf{A}_k = \{-100 + k, ..., 0, 1,..., 100 - k\}$, $k = 0, 1, 2, 3, ..., 100$, then simplify $\mathbf{A}_0\cap\mathbf{A}_1\cap...\cap\mathbf{A}_{100}$.

13. Show that the null set ϕ is a subset of all sets.

14. Show that

a. $(\mathbf{A}\cup\mathbf{B})'\cap\mathbf{B} = \phi$

b. $(\mathbf{A}\cap\mathbf{B})'\cup\mathbf{B} = \mathfrak{U}$

15. Let $\mathfrak{U} = \{0,1,2,3,...,10\}$, $\mathbf{A} = \{2,3,4,8,9\}$, $\mathbf{B} = \{1,3,5,7,9\}$, $\mathbf{C} = \{1,3,6,10\}$.

Simplify:

$\mathbf{E} = (\mathbf{A}\cap\mathbf{B}\cap\mathbf{C})\cup(\mathbf{A}\cap\mathbf{B}'\cap\mathbf{C})\cup(\mathbf{A}\cap\mathbf{B}\cap\mathbf{C}')\cup(\mathbf{A}\cap\mathbf{B}'\cap\mathbf{C}')\cup(\mathbf{A}'\cap\mathbf{B}\cap\mathbf{C})\cup(\mathbf{A}'\cap\mathbf{B}'\cap\mathbf{C})\cup(\mathbf{A}'\cap\mathbf{B}\cap\mathbf{C}')\cup(\mathbf{A}'\cap\mathbf{B}'\cap\mathbf{C}')$

16. Show the following are true:

a. $\mathbf{A}'' = \mathbf{A}$

DeMorgan's laws:

b. $(\mathbf{A}\cup\mathbf{B})' = \mathbf{A}'\cap\mathbf{B}'$

c. $(\mathbf{A}\cap\mathbf{B})' = \mathbf{A}'\cup\mathbf{B}'$

distributive laws:

d. $\mathbf{A}\cap(\mathbf{B}\cup\mathbf{C}) = (\mathbf{A}\cap\mathbf{B})\cup(\mathbf{A}\cap\mathbf{C})$

e. $\mathbf{A}\cup(\mathbf{B}\cap\mathbf{C}) = (\mathbf{A}\cup\mathbf{B})\cap(\mathbf{A}\cup\mathbf{C})$

f. $(\mathbf{A}\cup\mathbf{B})\cap(\mathbf{C}\cup\mathbf{D}) = (\mathbf{A}\cap\mathbf{C})\cup(\mathbf{A}\cap\mathbf{D})\cup(\mathbf{B}\cap\mathbf{C})\cup(\mathbf{B}\cap\mathbf{D})$